One-Loop Effects of a Heavy Higgs Boson: a Functional Approach*

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Abstract

We integrate out the Higgs boson in the electroweak standard model at one loop, assuming that it is very heavy. We construct a low-energy effective Lagrangian, which parametrizes the one-loop effects of the heavy Higgs boson at $\mathcal{O}(M_{\mathrm{H}}^0)$. Instead of applying conventional diagrammatical techniques, we integrate out the Higgs boson directly in the path integral.

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Introduction

Effective Lagrangians are used in order to describe the low-energy effects of heavy particles. An effective Lagrangian contains only light particles, and the heavy particles' effects are parametrized in terms of effective interactions of the light ones.

An effective Lagrangian can be constructed from the underlying theory by integrating out the heavy particles. This can be done in two different ways:

- *The diagrammatical method:* One calculates the relevant Feynman graphs with heavy particles and matches the full theory to the effective one.
- *The functional method:* One integrates out the the heavy fields directly in the path integral.

Here we focus on the functional method which turns out to be much more elegant and simpler than the diagrammatical one.

As a phenomenologically important application we consider the electroweak standard model (SM) provided that $M_{\mathrm{H}} \gg M_{\mathrm{W}}, E$. We integrate out the Higgs boson and determine the formal limit $M_{\mathrm{H}} \to \infty$ of the SM, i.e. all contributions of the Higgs boson to the resulting effective Lagrangian at $\mathcal{O}(M_{\mathrm{H}}^0)$ (which includes $\log M_{\mathrm{H}}$-terms).

In these proceedings we can sketch our method and our results only very briefly. The reader who is interested in a more detailed presentation is referred to the original articles Refs. [1, 2].

Integrating out the Higgs field

We briefly describe the basic concepts of our method to integrate out heavy fields in the path integral.

The background-field method

The SM Lagrangian contains terms cubic and quartic in the Higgs field. Thus, the integral over the Higgs field is not of Gaussian type. However, this problem can be circumvented by applying the background-field method (BFM) [3, 4], where each field is split into a (classical) background field and a quantum field, such that the functional integral is only performed over the latter. The background fields correspond at the diagrammatical level to tree lines while the quantum fields correspond to lines in loops. Thus, at one loop it is sufficient to consider only the part of the Lagrangian which is quadratic in the quantum fields. Then the heavy quantum field can be integrated out by Gaussian integration.

The Stueckelberg formalism

Another advantage of the use of the BFM is that different gauges may be chosen for the quantum and background fields, respectively [3, 4]. For our purposes it is useful to choose a generalized R_ξ-gauge [4] for the quantum fields (such that the quantized Lagrangian is still invariant under gauge transformations of the background fields [3, 4]) but the unitary gauge for the background fields. This aim can be achieved by applying a generalized [1, 2, 5] Stueckelberg transformation

[6, 7] to the (background and quantum) vector fields, which removes the background Goldstone fields from the Lagrangian. After all calculations are done, this transformation is inverted in order to reintroduce the background Goldstone fields and to obtain a manifestly gauge-invariant result.

Diagonalization of the Lagrangian

The one-loop Lagrangian of the SM (i.e. the part quadratic in the quantum fields) contains terms linear in the quantum Higgs field H. After Gaussian integration these would yield terms with inverse operators acting on the quantum fields. However, one can apply appropriate shifts to the quantum fields, such that these linear terms are removed while the Higgs-independent part of the Lagrangian remains unaffected [1, 5, 8]. The resulting Lagrangian can then be written in the symbolic form

$$\mathcal{L}^{1-\mathrm{loop}} = -\frac{1}{2} H \tilde{\Delta}_{H}(x, \partial_x) H + \mathcal{L}^{1-\mathrm{loop}}\Big|_{H=0}, \tag{1}$$

where the operator $\tilde{\Delta}_{H}$, which depends on the background fields, also contains contributions from the terms originally linear in the quantum Higgs field H owing to the shifts.

$1/M_{\mathrm{H}}$-expansion

Next the Gaussian integration over the quantum Higgs field can be performed. The resulting functional determinant can be parametrized in terms of an effective Lagrangian [1, 9]

$$\mathcal{L}_{\mathrm{eff}} = \frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \log\left(\tilde{\Delta}_{H}(x, \partial_x + ip)\right). \tag{2}$$

Then one can perform a Taylor expansion of the expression $\tilde{\Delta}_{H}(x, \partial_x + ip)$ around $\tilde{\Delta}_{H}(x, ip)$ (derivative expansion) followed by a Taylor expansion of the logarithm. These expansions yield one-loop vacuum integrals of the type

$$\int d^4p \frac{p_{\mu_1} \dots p_{\mu_{2k}}}{(p^2 - M_{\mathrm{H}}^2)^l (p^2 - \xi M_i^2)^m}, \qquad M_i = M_{\mathrm{W}}, M_{\mathrm{Z}}. \tag{3}$$

These are $\mathcal{O}(M_{\mathrm{H}}^0)$ or higher only for $4 + 2(k - l - m) \geq 0$. Thus, in the two above-mentioned Taylor expansions only a finite number of terms contribute to the effective Lagrangian $\mathcal{L}_{\mathrm{eff}}$ at $\mathcal{O}(M_{\mathrm{H}}^0)$.

Elimination of the background Higgs field

After the integration over the quantum Higgs field H, the effective Lagrangian still contains the background Higgs field $\hat{H}$. The latter corresponds to Higgs tree lines, and thus can easily be eliminated by a propagator expansion Diagrammatically this means that the $\hat{H}$ propagator shrinks to a point rendering such (sub-)graphs irreducible, which contain background Higgs lines only. Equivalently, the background Higgs-field can be eliminated by applying the classical equations of motion which are valid at tree level. Before eliminating the field $\hat{H}$, the Higgs sector has to be renormalized by adding the Higgs-dependent part of the counterterm Lagrangian.

The heavy-Higgs limit of the standard model

Proceeding as explained above, we find the formal limit of the SM for $M_{\rm H} \to \infty$ – i.e. the Lagrangian which contains the non-decoupling ($\mathcal{O}(M_{\rm H}^0)$) effects of the Higgs boson – at one loop [2]:

$$\mathcal{L}_{\rm SM}^{1-\rm loop}\Big|_{M_{\rm H}\to\infty} = \mathcal{L}_{\rm GNLSM}^{1-\rm loop} + \mathcal{L}_{\rm eff}. \tag{4}$$

In eq. (4) $\mathcal{L}_{GNLSM}^{1-loop}$ is the one-loop Lagrangian of the gauged nonlinear σ-model (GNLSM) [10], which is obtained from the SM Lagrangian by simply omitting the Higgs field in the unitary-gauge. $\mathcal{L}_{\rm eff}$ is the effective Lagrangian generated by integrating out the Higgs field and parametrizes the one-loop effects of the heavy Higgs field. Omitting terms which do not contribute to the S-matrix we find [2]

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{\rm S-matrix} \quad = \quad & \frac{1}{16\pi^2}\frac{3}{8}\Big(\Delta_{M_{\rm H}}+\frac{5}{6}\Big)\frac{g_1^2}{g_2^2}M_{\rm W}^2\left(\operatorname{tr}\left\{T\hat{V}_\mu\right\}\right)^2 \\
& -\frac{1}{16\pi^2}\frac{1}{24}\Big(\Delta_{M_{\rm H}}+\frac{5}{6}\Big)g_1g_2\hat{B}_{\mu\nu}\operatorname{tr}\left\{T\hat{W}^{\mu\nu}\right\} \\
& +\frac{1}{16\pi^2}\frac{1}{48}\Big(\Delta_{M_{\rm H}}+\frac{17}{6}\Big)ig_1\hat{B}_{\mu\nu}\operatorname{tr}\left\{T[\hat{V}^\mu,\hat{V}^\nu]\right\} \\
& -\frac{1}{16\pi^2}\frac{1}{24}\Big(\Delta_{M_{\rm H}}+\frac{17}{6}\Big)ig_2\operatorname{tr}\left\{\hat{W}_{\mu\nu}[\hat{V}^\mu,\hat{V}^\nu]\right\} \\
& -\frac{1}{16\pi^2}\frac{1}{12}\Big(\Delta_{M_{\rm H}}+\frac{17}{6}\Big)\left(\operatorname{tr}\left\{\hat{V}_\mu\hat{V}_\nu\right\}\right)^2 \\
& -\frac{1}{16\pi^2}\frac{1}{24}\Big(\Delta_{M_{\rm H}}+\frac{79}{3}-\frac{27\pi}{2\sqrt{3}}\Big)\left(\operatorname{tr}\left\{\hat{V}_\mu\hat{V}^\mu\right\}\right)^2+\mathcal{O}(M_{\rm H}^{-2})
\end{aligned} \tag{5}$$

with

$$\Delta_{M_{\rm H}} = \Delta - \log\left(\frac{M_{\rm H}^2}{\mu^2}\right), \qquad \Delta = \frac{2}{4-D} - \gamma_E + \log(4\pi), \tag{6}$$

where D is the space-time dimension in dimensional regularization, γ_E is Euler's constant, and μ is the reference mass.

In eq. (5) we have used the notation of Refs. [2, 10, 11], which we specify here only for the case of the U-gauge, where the background Goldstone fields are absent, and thus the physical content of the terms in $\mathcal{L}_{\rm eff}$ is most obvious:

$$\hat{V}^\mu = -\frac{i}{2}\left(g_2\hat{W}_i^\mu\tau_i + g_1\hat{B}^\mu\tau_3\right), \qquad T=\tau_3, \qquad \hat{W}^{\mu\nu} = \frac{1}{2}\hat{W}_i^{\mu\nu}\tau_i, \tag{7}$$

where $\hat{B}^\mu$ and $\hat{W}^\mu$ are the $U(1)$ and $SU(2)$ gauge fields, respectivly, g_1 and g_2 are the corresponding gauge couplings and the τ_i are the Pauli matrices. The hats over the fields indicate that these are background fields.

The result of our functional calculation agrees with the result of the diagrammatical calculation in Ref. [11].

The first two terms in eq. (5) contain vector-boson two-point (and higher) functions, the next two three-point (and higher) functions and the last two four-point functions. Thus, the first two parametrize the effects of the heavy Higgs boson to LEP 1 physics, the next two the effects to LEP 2 physics and the last two those to LHC physics.

$\mathcal{L}_{\rm eff}$ (5) does not contain custodial $SU(2)$ violating terms of dimension 4, although there are 7 such terms which are by naive power counting expected to be generated when integrating out the Higgs field [10, 11]. As shown in Ref. [2], within our functional calculation it is obvious that these terms vanish (while in a diagrammatical calculation [11] their absence seems to be an accidental cancellation).

The effective interaction terms in eq. (5) have logarithmically divergent coefficients. Owing to the renormalizability of the SM, these UV-divergences cancel against the logarithmically divergent one-loop contributions from the GNLSM Lagrangian in eq. (4) [10]. Since logarithmic divergences and $\log M_{\rm H}$-terms in $\mathcal{L}_{\rm eff}$ always occur in the combination $\Delta_{M_{\rm H}}$ (6), the $\log M_{\rm H}$-terms in the SM coincide with the logarithmically divergent terms in the GNLSM, as assumed in Ref. [10]. However, in addition eq. (5) contains finite and constant differences between the SM and the GNLSM at one loop.

Conclusion

Our purpose with this project is twofold: On the one hand, we integrate out the Higgs boson in the electroweak standard model at one loop. We parametrize the non-decoupling (i.e. $\mathcal{O}(M_{\rm H}^0)$) effects of this field in terms of a low-energy effective Lagrangian. On the other hand, we have developed a functional method to integrate out heavy fields directly in the path integral. This method can be applied to integrate out any kind of non-decoupling heavy field and also be generalized to a decoupling scenario.

Our method is an alternative to the conventional diagrammatical techniques and turns out to be a huge simplification. While in a diagrammatical calculation various Green functions (i.e. very many Feynman diagrams) have to be calculated and the effective Lagrangian has to be determined indirectly by matching the full theory to the effective one [11], in a functional calculation the effective Lagrangian is generated *directly* by integrating out the heavy fields. The use of the BFM and of the Stueckelberg formalism automatically ensures gauge invariance of the result.

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